

# Dynamical Aspects of P Systems

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## Abstract

A dynamical analysis of P systems is given that is focused on basic phenomena of biological relevance. After a short presentation of a new kind of P systems, called PB Systems, membrane systems with environment, called PBE Systems, are introduced that are more suitable for modeling complex membrane interactions. Some types of periodicity and non-periodicity are considered for PBE systems by showing some “minimal” examples of systems that exhibit these properties. In particular, a discrete formulation of the Belousov-Zhabotinsky reaction is given in terms of PBE systems. Some questions and open problems for future research are indicated.

**Key-words:** P Systems, Membrane Systems, Dynamical Systems, Biological Models, Periodicity, Almost Periodicity.

## 1 Introduction

P Systems are a class of distributed and parallel computing devices of a biological type introduced by Păun (2000, 2001). The basic model consists of a structure composed by several membranes hierarchically embedded in a main membrane. Each membrane delimits a “region” and contains a multiset of objects. The objects evolve according to given rules associated with regions. A rule can modify the objects and send them outside the membrane or into an inner membrane.

Communication of objects through membranes is one of the most important ingredients of P systems and recently, in Păun (2002), a biochemical transport mechanism was introduced that is based on pairs of chemicals called *symport/antiport*. A natural generalization of this mechanism was considered by the authors Bernardini (2002), Bernardini and Manca (2002) where the key point is the notion of *boundary rule*. This means that rules are not internal to a region, but rather they are able to *see* even externally. In other words, a boundary rule is sensible to what happens around its border. This perspective is closer to the

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biological reality, and moreover has the advantage to simplify many notational aspects and to eliminate a certain level of asymmetry implicit in P systems where membranes are passive entities.

P Systems with boundary rules, shortly PB systems, strengthen the role of membranes providing a mechanism to define more complex interaction patterns among membranes. For this reason it seems us that PB systems could provide an adequate basis for a new kind of investigations on membrane systems.

In fact, in this paper, starting from P systems with boundary rules, we introduce an approach to the study of P systems that is devoted to typical dynamical aspects of biological systems:

- *periodicity and quasi-periodicity.* Life is always related to temporal cycles where, even if some temporal irreversibility is intrinsic, many parameters change periodically and some basic rhythms are preserved. This means that clocks are necessary for life and especially organization and harmonization of clocks are essential in developing complex behavioral patterns.
- *stability and adaptability.* Biological systems tend, within some limits, to keep their “form” and their basic “behavior” even if their external world changes.
- *growth and degeneration.* A living organism is able to survive when it maintains in time some basic oscillating reactions. Three main tasks are intrinsic to its life: growing, reproducing, and dying. They are very complex inasmuch as are coupled with linear oriented processes.
- *reproduction and evolution.* This is the last level of the organization of living organisms, where the dynamics is concerned with the passage from ‘instances’ of life to ‘forms’ of life.

In this paper we deal with the aspect of periodicity. Our interest is in the individuation of “minimal” PB systems that exhibit interesting phenomena of periodicity in a given environment. To this end, it is necessary to start with a formal definition of environment for a PB system, and to define some kinds of periodicity and non-periodicity. Then, some PB systems are considered and for each of them some specific properties are showed. We remark that the active role of membrane in the communication with external regions is a crucial point of our formal model.

The study of interactions of two or more oscillating systems is directly connected with biological problems. For this reason, the coupling of chemical oscillators has been the subject of a number of studies. A simple example of such an oscillation is the famous Belousov-Zhabotinsky (BZ) reaction, a metal ion-catalyzed oxidation and bromination of an organic substrate (Zhabotinsky, 1991). It turns out that some discrete symbolic formulation of BZ reaction (Suzuki and Tanaka, 1997) are particular cases of PB systems with environment, shortly PBE systems.

Our investigation is a preliminary step. Many questions and many aspects deserve a better and deeper analysis. However, it seems us that the introduction

of this problematic, in a field based on discrete and symbolic concepts, could disclose new possibilities in the formalization of basic aspects of life.

## 2 Preliminaries

We recall some basic notions concerning formal language theory and multisets which will be used in this paper; for formal language theory we refer to the handbook of Rozenberg and Salomaa (1998) while, for multisets we refer to Păun (2001) and Păun (2002).

An *alphabet* is a finite non-empty set of abstract symbols. Given an alphabet  $V$  we denote by  $V^*$  the set of all possible strings over  $V$ , including the empty string  $\lambda$ . The length of a string  $x \in V^*$  is denoted by  $|x|$  and  $|x|_a$ , for  $a \in V$ , is the number of occurrences of  $a$  in  $x$ .

A multiset over  $V$  is a mapping  $M : V \rightarrow \mathbb{N}$  such that,  $M(a)$  defines the *multiplicity of  $a$  in the multiset  $M$*  ( $\mathbb{N}$  is the set of natural numbers); the symbols in  $V$  are called *objects*. Such a multiset can be represented by a string  $a_1^{M(a_1)} a_2^{M(a_2)} \dots a_n^{M(a_n)} \in V^*$  and by all its permutations, where  $a_j \in V$ ,  $M(a_j) \neq 0$ ,  $1 \leq j \leq n$ . In other words, we can say that each string  $x \in V^*$  identifies a finite multiset over  $V$  defined by  $m(x) = \{(a, |x|_a) \mid a \in V\}$ . The *union* of two multisets represented by two strings  $x, y \in V^*$  is the multiset  $m(x) \sqcup m(y) = m(xy)$ , where  $xy$  is the concatenation of  $x$  and  $y$ . We say that a multiset represented by a string  $x$  is *included* in a multiset represented by string  $y$ . We write  $m(y) \supseteq m(x)$  if  $x'$  is a substring of  $y$ , for some permutation  $x'$  of  $x$  ( $\supseteq$  being the strict inclusion that excludes equality). The *size* of a multiset represented by a string  $x$  is defined by  $size(m(x)) = |x|$ .

In the sequel, in order to simplify many formulations, we will use interchangeably the expressions “multiset represented by a string  $x$ ”, “multiset  $x$ ” and “string  $x$ ”.

An *infinite sequence of multisets*  $\Sigma$  is an infinite set of multisets (strings) with an index, that is,  $\Sigma = \{\alpha_j \mid j \geq 1\}$ . Given an infinite sequence of multisets  $\Sigma = \{\alpha_j \mid j \geq 1\}$  and two indexes  $0 \leq j_1 \leq j_2$  we define the function:

$$C_\Sigma(j_1, j_2) = \bigsqcup_{j_1 \leq j \leq j_2} \alpha_j.$$

That is,  $C_\Sigma(j_1, j_2)$  is the function which “collects” the objects which appear in the sequence from the index  $j_1$  to the index  $j_2$ .

We will give some definitions which characterize infinite sequences of multisets in terms of particular properties of “regularity”. The search for “regularity” in sequences of multisets is related to the more general problem of classifying the “behavioral patterns” that are exhibited by P systems. In the next sections we will discuss some cases.

**Definition 2.1.** Let  $\Sigma = \{\alpha_j \mid j \geq 1\}$  be an infinite sequence of multisets. We say  $\Sigma$  is *periodic* if exist  $k_0 \geq 0$ ,  $k > 0$  and some multisets  $\beta_1, \beta_2, \dots, \beta_k$  such

that, for all  $n \geq 0$ ,  $1 \leq j \leq k$ , we have:

$$C_{\Sigma}(k_0 + nk + j, k_0 + nk + j) = \alpha_{k_0 + nk + j} = \beta_j \quad (1)$$

The minimum  $k$  and the minimum  $k_0$  which satisfy the condition (1) are respectively called the *period* of  $\Sigma$  and the *transient* of  $\Sigma$ .

**Definition 2.2.** Let  $\Sigma = \{\alpha_j \mid j \geq 1\}$  be an infinite sequence of multisets. We say that  $\Sigma$  is *almost periodic* if exist  $k_0 \geq 0$ ,  $k > 0$  and a multiset  $\beta$  such that, for all  $n \geq 0$ , we have:

$$C_{\Sigma}(k_0 + nk + 1, k_0 + nk + k) \supseteq \beta \quad (2)$$

The minimum  $k$  and the minimum  $k_0$  which satisfy the condition (2) are respectively called the *period* of  $\Sigma$  and the *transient* of  $\Sigma$ .

The condition (2) is essentially the property of almost periodicity of (infinite) strings of Marcus and Păun (1994). In fact, if  $\gamma$  is the (infinite) string obtained by concatenating  $\alpha_j$ , for  $j > 0$ , then  $\gamma$  can be factorized in substring of equal length  $k$  such that each of them includes the string  $\beta$ .

**Definition 2.3.** Let  $\Sigma = \{\alpha_j \mid j \geq 1\}$  be an infinite sequence of multisets. We say that  $\Sigma$  is *growing almost periodic* if exist  $k_0 \geq 0$ ,  $k > 0$  and some multisets  $\beta_1, \beta_2, \dots, \beta_k$  such that, for all  $n \geq 0$ ,  $1 \leq j \leq k$ , we have:

$$C_{\Sigma}(s_{n+1}, s_{n+1}) \supseteq C_{\Sigma}(s_n, s_n) \supseteq \beta_j \quad (3)$$

with  $s_{n+1} = k_0 + (n+1)k + j$  and  $s_n = k_0 + nk + j$ . The minimum  $k$  and the minimum  $k_0$  which satisfy the condition (3) are respectively called the *period* of  $\Sigma$  and the *transient* of  $\Sigma$ .

Clearly, every growing almost periodic sequence of period  $k$  is almost periodic of period  $k$ , for  $\beta = \beta_1 \sqcup \beta_1 \sqcup \dots \sqcup \beta_k$ .

### 3 P Systems with boundary rules

In this section we introduce the class of *P Systems with Boundary rules* (for short *PB Systems*), a variant of P system proposed in Bernardini (2002), Bernardini and Manca (2002).

A membrane structure is a construct consisting of several membranes placed in a unique membrane called *skin membrane*. As usual, a membrane structure is represented by a string of correctly matching parentheses. Then, if we place a multiset on the right side of each open parentheses, we obtain a *configuration* of a PB system represented by string like this:

$$[1 \ a \ b [2 \ a \ b]_2 [3 \ [4 \ c \ d]_4 [5 \ a \ c]_5]_3]_1.$$

Membranes are labeled, in one-to-one manner, by values in  $\{1, 2, \dots, m\}$ . The label of the skin membrane is assumed to be 1.

Now we are ready to define P Systems with Boundary rules (for short, PB Systems):

**Definition 3.1.** A *P System with Boundary rules* is a construct:

$$\Pi = (V, \mu_0, R, i_O)$$

where:

- (i)  $V$  is an *alphabet* of symbols;
- (ii)  $\mu_0$  is the *initial* configuration;
- (iii)  $R$  is a *finite set of rules* of the following forms:

- $xx' [i y'y \rightarrow xy' [i x'y$  for  $x, y, x', y' \in V^*$  and  $1 \leq i \leq m$  (*Communication rules*);
- $[i y \rightarrow [i y'$  for  $y, y' \in V^*$  and  $1 \leq i \leq m$  (*Transformation rules*);

- (iv)  $i_O \in \{1, \dots, m\}$  is the label of the *output membrane*.

In this definition we can find the basic elements of every membrane systems: a structure which contains some multisets of objects (the configuration  $\mu_0$ ), a finite set of rules  $R$ , and an output membrane  $i_O$ .

In the set  $R$  we identify two types of rules: communication rules and transformation rules. A transformation rule  $[i y \rightarrow [i y'$  allows us to produce, in the membrane  $i$ , a new multiset  $y'$  starting from the multiset  $y$ ; as usual, the application of a transformation rule “consumes” the objects in  $y$ . Instead, with communication rules of the form  $xx' [i y'y \rightarrow xy' [i x'y$  we can move objects through membranes: if the membrane  $i$  contains the multiset  $y'y$  and outside the membrane  $i$  is present the multiset  $xx'$ , the multiset  $x'$  moves into the membrane  $i$  and the multiset  $y'$  is sent out from it; clearly, some of these multisets may be empty.

As usual, the rules are applied in a nondeterministic and maximally parallel manner: the rules are chosen in a nondeterministic manner and this choice is “exhaustive” because no other rule can be added to them.

A *computation* in a PB system is a sequence of transitions between configurations of the form:  $\mu_0 \Rightarrow \mu_1 \Rightarrow \mu_2 \Rightarrow \dots$  where, for all  $j > 0$ ,  $\mu_j$  is the configuration obtained by applying rules of  $R$  to the configuration  $\mu_{j-1}$ . A computation is successful if it *halts* with a configuration where no rules can be applied. The *result* of a successful computation is the multiset contained in the membrane  $i_O$  in the halting configuration. A computation which never halts yields no result.

In Bernardini (2002) the computational universality of P systems with boundary rules was proved by showing that PB systems with only three membranes are able to characterize the recursively enumerable sets of vectors of natural numbers.

## 4 PB Systems with environment

In very simple terms, we can think an environment as a multiset of objects placed outside the membrane structure defining the system. However, in order to get a more realistic representation, we will define an environment as a sequence of multisets, rather than a single multiset of objects. In other words, we will consider environments which could modify its state in time and where some objects appear only at specified times. This corresponds to the idea of an environment which cyclically provides some substances indispensable to ensure the “life” of the system.

In particular, we introduce the notion of *environment cycle of period  $k$* : the infinite sequence where  $k$  multisets, with  $k > 0$ ,  $\beta_0, \beta_1, \dots, \beta_{k-1}$  occur periodically in time. More precisely, we assume a global clock where instants coincide with the indexes of the sequence and we suppose that the sequence  $\beta_0, \beta_1, \dots, \beta_{k-1}$  was cyclically produced in the environment.

Now we define PB Systems with environment (in short PBE Systems) in the following manner:

**Definition 4.1.** A PB System with environment (PBE System) is a construct:

$$\Pi = (V, \mu_0, R, E, R_E, O)$$

where:  $V, \mu_0, R$  are as in definition 3.1,  $E$  is an *environment cycle of period  $k$* ,  $R_E$ , the *environment rules*, is a finite set of rewriting rules on multisets of the form  $x \rightarrow y$ , for  $x, y \in V^*$  and  $O$  is the label of the *observable membrane*.

Now, the behavior of a PBE system  $\Pi$  can be described by sequences of (global) configurations:

$$\eta_0 \Rightarrow \eta_1 \Rightarrow \eta_2 \Rightarrow \dots \quad (4)$$

where  $\eta_0 = \beta_0 \mu_0$ , and  $\beta_0 \in E$  is the first multiset of the environment cycle,  $\mu_0$  is the initial (internal) configuration of  $\Pi$  and, for all  $j > 0$ ,  $\eta_j = \beta_j \gamma_j \mu_j$  with:

- $\beta_j \gamma_j$  the multiset which defines the environment configuration at time  $j$ ; it is formed by:  $\beta_j$  the multiset produced by the environment cycle at time  $j$  and  $\gamma_j$  the multiset obtained by applying both rules of  $R_E$  and rules of  $R$  to the environment configuration  $\beta_{j-1} \gamma_{j-1}$  (being  $\gamma_0$  the empty multiset);
- $\mu_j$ , the (internal) configuration obtained by applying the rule in  $R$  to the global configuration  $\beta_{j-1} \gamma_{j-1} \mu_{j-1}$ .

We remark that the sequence  $\{\beta_t \gamma_t \mid t \geq 0\}$  is an almost-periodic infinite sequence of multisets according to definition 2.2.

We “observe” the contents of the membrane labeled by  $O$ . More precisely, given a sequence of transition as (4), the behavior of a PBE system  $\Pi$  with respect to the observable  $O$  is represented by the sequence:

$$\Omega_O(\eta_0), \Omega_O(\eta_1), \Omega_O(\eta_2), \dots$$

where  $\Omega_O(\eta_j)$ , for  $j \geq 0$ , denotes the multiset contained in the membrane labeled by  $O$ , in the configuration  $\eta_j$ ; the multiset  $\Omega_O(\eta_j)$  is called *the value of the observable  $O$  in the configuration  $\eta_j$* ; we also say that the PBE system  $\Pi$  generates the *observable sequence*  $\Sigma_O = \{\Omega_O(\eta_j) \mid j \geq 0\}$ .

The role of the environment in PBE systems, according to the previous definition, resembles very much the symport/antiport mechanism provided by Paun and Paun (2002) whereby the environment is an active partner supplying the membrane system with as many copies of each object as it needs. Moreover, the interaction of PBE systems with the environment is very similar to the basic mechanism of stream X-machines (Eilenberg Machines) and their generalizations (Gheorghe 2000). However, what is peculiar of PBE systems is the assumption of a periodic behavior of the environment with respect to which the corresponding behavior of a system is considered. This suggests possible elaborations on the relationships between PBE systems with symport/antiport P systems and with stream X-machines.

## 5 PBE Systems with resources

In order to specify in a more precise way the role of the environment, we are going to consider a particular class of PBE systems where both communication and transformation rules need *resources*, that is, objects which are consumed when the rules are applied. Resources are the “fuel” produced by the environment which allow the system to “work” further on.

More precisely, we represent the resources used by the rules as symbols of a finite set  $\{r_i \mid 1 \leq i \leq h\}$ , for a given  $h$ ; each  $r_i$  denotes a different kind of resources. Then, we consider PBE systems such that the rules in  $R$  have the following forms:

- $xx' [i y' y r_j^k \rightarrow xy' [i x' y$  for  $x, y, x', y' \in V^*$ ,  $1 \leq j \leq h$ ,  $k > 0$  and  $1 \leq i \leq m$ ; (*Communication rules*);
- $[i y r_j^k \rightarrow [i y'$  for  $y, y' \in V^*$ ,  $1 \leq j \leq h$ ,  $k > 0$  and  $1 \leq i \leq m$  (*Transformation rules*);

Moreover, we consider transformation rules which produce some “waste objects” denoted by  $\#_t$ , for  $1 \leq t \leq w$ . The waste objects and the resources are respectively moved through membranes using communication rules of the form  $[i \#_t \rightarrow \#_t [i, r_j [i \rightarrow [i r_j$ .

Waste objects and resources are objects with an intrinsic direction: the former ones are expelled from the system, while the latter ones are acquired by the system (from the environment). In a next example we show a case where waste objects, after their expulsion, are transformed by the environment in resources, by using some other resources. This is the basic mechanism of many life cycles.

In the following examples, for the sake of simplicity, we will consider only deterministic PBE systems, that is, PBE systems where, in each step, there is

only one set of applicable rules; to this end, we suppose that each rule uses a different kind of resources.

**Example 5.1.** Here we model a simple interplay between two membranes. Membrane 1 acquires from the environment some resources  $r_1$  and uses them to produce an object of type  $b$  starting from an object of type  $a$  which remains in membrane 1. Then, the objects of type  $b$  are moved into membrane 2 using some resource  $r_2$ . We assume that the environment cycle provides two copies of  $r_1$  and  $r_2$ , therefore at end of the cycle, two copies of  $b$  will get the membrane 2. This idea is formalized by the following PBE system:

$$\Pi_1 = (V, \mu_0, R, E, R_E, O)$$

where

$$\begin{aligned} V &= \{a, b, r_1, r_2\} \\ \mu_0 &= [1 a [2 ]_2 ]_1 \\ R &= \{[1 a r_1 \rightarrow [1 a b ; b [2 r_2 \rightarrow [2 b ; r_1 [1 \rightarrow [1 r_1 ; r_2 [1 \rightarrow [1 r_2 ; r_2 [2 \rightarrow [2 r_2 ]\} \\ E &= \{r_1, r_1, r_2^2, \lambda, \lambda\} \\ R_E &= \emptyset \\ O &= 1 \end{aligned}$$

The behavior of this PBE system is described by the following sequence of transitions:

$$\begin{aligned} \eta_0 = r_1 [1 a [2 ]_2 ]_1 &\Rightarrow r_1 [1 r_1 a [2 ]_2 ]_1 \\ &\Rightarrow r_2^2 [1 r_1 a b [2 ]_2 ]_1 \\ &\Rightarrow [1 r_2^2 a b b [2 ]_2 ]_1 \\ &\Rightarrow [1 a b b [2 r_2^2 ]_2 ]_1 \\ &\Rightarrow \eta_1 = r_1 [1 a [2 b b ]_2 ]_1 \\ &\Rightarrow r_1 [1 r_1 a [2 b b ]_2 ]_1 \\ &\vdots \end{aligned}$$

In this case, if we observe the configuration  $\eta_1$ , we note that the observable membrane assumes again the initial value and we can apply the same set of rules. Thus, the system will continue to repeat cyclically the same sequence of transitions.

The following proposition establishes the periodic behavior of the previous system.

**Proposition 5.2.** *The observable sequence  $\Sigma_1$  generated by the PBE system  $\Pi_1$  is periodic, according to definition 2.1, with transient  $k_0 = 0$  and period  $k = 5$ .*



**Proof.** Consider the starting configuration  $\eta_0$  and the configuration  $\eta_1$ . We observe that  $\eta_0, \eta_1$  have the same pattern which can be represented by a string of the form

$$r_1 [{}_1 a [{}_2 b^n]_2]_1 \quad (5)$$

for  $n \geq 0$ . If we apply the rules of  $\Pi_1$  to the configuration (5), we obtain the following sequence of transitions:

$$\begin{aligned} r_1 [{}_1 a [{}_2 b^n]_2]_1 &\Rightarrow r_1 [{}_1 r_1 a [{}_2 b^n]_2]_1 \\ &\Rightarrow r_2^2 [{}_1 r_1 a b [{}_2 b^n]_2]_1 \\ &\Rightarrow [{}_1 r_2^2 a b b [{}_2 b^n]_2]_1 \\ &\Rightarrow [{}_1 a b b [{}_2 r_2^2 b^n]_2]_1 \\ &\Rightarrow r_1 [{}_1 a [{}_2 b b^{n+2}]_2]_1 \end{aligned}$$

At this point, setting  $n' = n + 2$ , we obtain a configuration like (5) and we can repeat the same sequence of transitions. Therefore, starting from  $\eta_0$ , the PBE system  $\Pi_1$  cyclically produces, in the membrane 1, the sequence of values  $a, a, ab, abb, abb$ . That means, the observable sequence  $\Sigma_1$  is periodic, according to definition 2.1, with transient  $k_0 = 0$  and period  $k = 5$ .

Now we illustrate an example of PBE system which exhibits a non periodic behavior.

**Example 5.3.** We consider another interplay between two membranes which models a sort of “metabolic cycle”. In membrane 1, starting from an object  $a$  and a resource  $r_1$ , we produce a copy of  $a$ , a copy of  $b$  and a waste object  $\#_1$ ; the object  $\#_1$  is immediately released in the environment while, the objects  $b$  is moved into membrane 2 using a resource  $r_2$ . In the membrane 2, using a resource  $r_3$ , the object  $b$  is transformed in a waste object  $\#_2$  which, in 2 steps, is released in the environment. The environment cycle produces, with a certain frequency, objects of type  $a$  and objects of type  $b$ . When both a copy of  $a$  and a copy of  $\#_1$  are present in the environment a new copy of  $r_1$  is produced; when both a copy of  $b$  and a copy of  $\#_2$  are present in the environment a new copy of  $r_2$  and  $r_3$  is produced. The resources  $r_1, r_2, r_3$  are immediately acquired from the skin membrane and the cycle starts again. Formally, we define the following PBE system:

$$\Pi_2 = (V, \mu_0, R, E, R_E, O)$$

where

$$\begin{aligned}
V &= \{a, b, r_1, r_2, r_3, \#_1, \#_2\} \\
\mu_0 &= [{}_1 r_1 r_2 r_3 a [{}_2 ]_2 ]_1 \\
R &= \{[{}_1 r_1 a \rightarrow [{}_1 ab\#_1; b [{}_2 r_2 \rightarrow [{}_2 b; [{}_1 \#_1 \rightarrow \#_1 [{}_1 ; \\
&\quad [{}_2 r_3 b \rightarrow [{}_2 \#_2; [{}_2 \#_2 \rightarrow \#_2 [{}_2 ; [{}_1 \#_2 \rightarrow \#_2 [{}_1 ]\} \\
&\cup \{r_i [{}_1 \rightarrow [{}_1 r_i \mid 1 \leq i \leq 3\} \\
&\cup \{r_i [{}_2 \rightarrow [{}_2 r_i \mid 2 \leq i \leq 3\} \\
E &= \{\lambda, \lambda, \lambda, a, b, \lambda\} \\
R_E &= \{a\#_1 \rightarrow r_1; b\#_2 \rightarrow r_2 r_3\} \\
O &= 1
\end{aligned}$$

In this case, if we generate the sequence of transitions which starts from the initial configuration, we note that, after 30 steps, the system reaches the configuration  $\eta_1 = b [{}_1 abb\#_1 [{}_2 r_2 r_3 ]_2 ]_1$ , then, after 42 more steps, it reaches the configuration  $\eta_2 = b [{}_1 abbb\#_1 [{}_2 r_2 r_3 ]_2 ]_1$ , after 42 more steps, the configuration  $\eta_3 = b [{}_1 abbbb\#_1 [{}_2 r_2 r_3 ]_2 ]_1$  and so on. In the configurations  $\eta_1, \eta_2, \eta_3$  the set of applicable rules is the same, thus, the PBE system  $\Pi_2$ , after 30 steps, continues to repeat the same sequence of transitions which, every 42 steps, adds a  $b$  to the membrane 1. More precisely, the following proposition holds.

**Proposition 5.4.** *The observable sequence  $\Sigma_1$  generated by  $\Pi_2$  is growing almost periodic, according to definition 2.3, with transient  $k_0 = 30$  and  $k = 42$ .*

**Proof.** Consider the configurations  $\eta_1, \eta_2, \eta_3$  obtained as above. Such configurations have the same pattern which can be represented by a string of the form

$$b [{}_1 ab^n \#_1 [{}_2 r_2 r_3 ]_2 ]_1 \quad (6)$$

for  $n \geq 2$ . If we apply the rules to the configurations (6) we obtain, in membrane 1, the following sequence of values:

$$42 \text{ steps } \begin{cases} ab^n, ab^{n-1}, ab^{n-1}, ab^{n-1}, ab^{n-1}, ab^{n-1}, \\ ab^n, ab^n, ab^{n-1}, ab^{n-1}, ab^{n-1}, ab^{n-1}, \\ ab^n, ab^n, ab^n, ab^{n-1}, ab^{n-1}, ab^{n-1}, \\ ab^n, ab^n, ab^n, ab^n, ab^{n-1}, ab^{n-1}, \\ ab^n, ab^n, ab^n, ab^n, ab^n, ab^{n-1}, \\ ab^n, ab^n, ab^n, ab^n, ab^n, ab^n, \\ ab^n, ab^n, ab^n, ab^n, ab^n, ab^n, \end{cases} \quad (7)$$

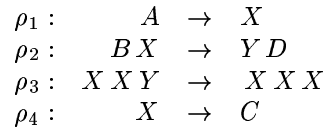
Then, in the next step, the system reaches the configuration  $b [{}_1 ab^{n+1} \#_1 [{}_2 r_2 r_3 ]_2 ]_1$ ; at that point, if we set  $n' = n + 1$ , in 42 more steps, we produce, in membrane 1, a sequence as (7). Thus the condition (3) of definition 2.3 holds for  $k_0 = 30$ ,  $k = 42$  and for a sequence of 42 multisets with the same structure of the sequence (7) where  $n = 1$ . Moreover,  $k_0 = 30$  and  $k = 42$  are the minimum values which satisfy the condition (3).

Another interesting property which can be associated with the PBE System  $\Pi_2$  is *non creativity*: all the rules, both those in  $R$  and those in  $R_E$ , “consume” some objects and all the objects “produced” by some rules are consumed by some other rules. In other words, a non creative system defines a “cycle” where no new objects are “created” but where every object is “transformed” in another one. The only new objects that are introduced in the system are provided by the environment cycle; such objects represent the “fuel” the system needs in order to keep working.

In the case of the PBE system  $\Pi_2$  of the example 5.3, as we have shown, the system takes some advantage with respect to the environment cycle, in such a way that the  $b$  are provided by the environment more frequently than the system can consume them. This will increase, in the time, the amount of  $b$  inside the system. Therefore, though the system  $\Pi_2$  is non creative (as it results after a check of its rules), nevertheless it exhibits a non periodic behavior because it does not consumes too much.

## 6 The Brusselator

The Brusselator is a model of chemical oscillations based on the famous Belousov-Zhabotinsky reaction that is described in Zhabotinski (1991). In Suzuki and Tanaka (1997), a formulation of such a reaction is given in terms of an abstract rewriting system on multisets (ARM) that is based on the following set of rules:



The rule  $r_1$  produces objects of type  $X$ ; the rule  $r_2$  consumes  $X$  and produces objects of type  $Y$ ; the rule  $r_3$  uses a copy of  $Y$  for increasing the number of  $X$ . Finally, the rule  $r_4$  allows the system to consume the objects  $X$ .

In Suzuki and Tanaka (1997) the behavior of this rewriting system was investigated by observing the relationship between the frequency of rule applications and the concentration of  $X$  and  $Y$ , assuming that  $A$  and  $B$  were continually provided to the system from the external world. The frequency of rule applications corresponds to the chemical “reaction rates”.

This idea of Brusselator can be formulated in terms of PBE system in a very natural way considering  $r_1, r_2, r_3, r_4$  as resources which the rules  $\rho_1, \rho_2, \rho_3, \rho_4$  use respectively for their application. We assume that objects  $r_1, r_2, r_3, r_4$  are produced by the environment cycle with a certain frequency. More precisely, we define the following PBE system:

$$\Pi_{BZ} = (V, \mu_0, R, E, R_E, O)$$

where

$$\begin{aligned}
V &= \{A, B, C, D, X, Y, r_1, r_2, r_3, r_4\} \\
\mu_0 &= [1]_1 \\
R &= \{[1 r_1 A \rightarrow [1 X]; [1 r_2 B X \rightarrow [1 Y D]; [1 r_3 X X Y \rightarrow [1 X X X]; \\
&\quad [1 r_4 X \rightarrow [1 C]; A [1 \rightarrow [1 A]; B [1 \rightarrow [1 B]\} \\
&\cup \{r_i [1 \rightarrow [1 r_i \mid 1 \leq i \leq 4]\} \\
E &= \{\beta_1, \beta_2, \dots, \beta_k\} \text{ for } k > 0, \beta_i = A B r_j, 1 \leq i \leq k, 1 \leq j \leq 4 \\
R_E &= \emptyset \\
O &= 1
\end{aligned}$$

In this formulation, the environment cycle  $E$  produces a sequence of multisets of the form  $AB r_j$ , for  $1 \leq j \leq 4$ , that is, the environment cycle, at every step, produces a copy of  $A$ , a copy of  $B$ , and an object  $r_j$  which identifies the rule which will be applied in the next step. Therefore, for all  $1 \leq j \leq 4$ , the number of copies of  $r_j$  in the sequence  $E$  defines the frequency of application of the rule  $\rho_j$ .

Now we pass to consider an example of Brusselator in terms of PBE system.

**Example 6.1.** Consider the PBE system  $\Pi_{BZ}$  as defined above where:

$$\begin{aligned}
E &= \{AB r_1, AB r_1, AB r_1, AB r_1, AB r_2, AB r_2, AB r_3, AB r_3, \\
&\quad AB r_4, AB r_4, AB r_4, AB r_4\}
\end{aligned}$$

In this case the behavior of  $\Pi_B$  can be described by the following sequence of transitions:

$$\begin{aligned}
AB r_1 [1]_1 &\Rightarrow \eta_1 = AB r_1 [1 AB r_1]_1 \\
&\Rightarrow AB r_1 [1 A r_1 X B^2]_1 \\
&\Rightarrow AB r_1 [1 A r_1 X X B^3]_1 \\
&\Rightarrow AB r_2 [1 A r_1 X X X B^4]_1 \\
&\Rightarrow AB r_2 [1 A r_2 X X X X B^5]_1 \\
&\Rightarrow AB r_3 [1 A^2 r_2 X X X Y B^5 D]_1 \\
&\Rightarrow AB r_3 [1 A^3 r_3 X X Y Y B^5 D^2]_1 \\
&\Rightarrow AB r_4 [1 A^4 r_3 X X X Y B^6 D^2]_1 \\
&\Rightarrow AB r_4 [1 A^5 r_4 X X X X B^7 D^2]_1 \\
&\Rightarrow AB r_4 [1 A^6 r_4 X X X B^8 D^2 C]_1 \\
&\Rightarrow AB r_4 [1 A^7 r_4 X X B^9 D^2 C^2]_1 \\
&\Rightarrow AB r_1 [1 A^8 r_4 X B^{10} D^2 C^3]_1 \\
&\Rightarrow \eta_2 = AB r_1 [1 A^9 r_1 B^{11} D^2 C^4]_1 \\
&\Rightarrow AB r_1 [1 A^9 r_1 X B^{12} D^2 C^4]_1 \\
&\Rightarrow AB r_1 [1 A^9 r_1 X X B^{13} D^2 C^4]_1 \\
&\vdots
\end{aligned}$$

If we observe the concentration of  $X, Y$  in the membrane, we note that in the configuration  $\eta_1$  the number of  $X$  and  $Y$  is the same as in configuration  $\eta_2$  and in these configurations the set of applicable rule is the same. Thus, the observable sequence of  $X, Y$  that is generated by the PBE system  $\Pi_{BZ}$  is periodic (it could be proved as in proposition 5.2). Moreover, we observe that the number of  $X$  is initially increased by using  $r_1$ , while this number is decreased by using  $r_2$  when objects of type  $Y$  are produced, but the number of  $X$  is increased again by using  $r_3$ . Finally, the membrane is emptied, by using  $r_4$ , and the cycle starts again. This confirms the stable oscillations of the concentration of  $X, Y$  observed in Suzuki and Tanaka (1997), under certain conditions.

The previous example shows a particular case where periodicity can be formally derived under some specific assumptions. However, it would be very interesting to find general conditions, based on the frequencies of the rules and on some regulatory mechanisms, that ensure some kind of periodicity with a reasonable adequacy to the real chemical dynamics.

In fact, rather than applying rules in a nondeterministic and maximally parallel way, some “activation conditions” should determine a “percentage of applicability”, for each rule, that depends on the concentration of some kind of objects.

## 7 Conclusions

Life is a very complex phenomenon that is possible due to the cooperation of many interacting processes. The search for laws expressing the principles that rule the regularity of some biological dynamics is very often unsuccessful if we use the “glasses” of classical mathematics.

Discrete symbolic models, naturally based on non-quantitative aspects, could provide new formal methods in modeling biological phenomena. In the context of automata theory, in von Neumann (1966) a model was proposed of self-reproducing machines, called *cellular automata* that have been extensively studied in a spirit closer to dynamical systems, rather than to the formal language theory (Bonanno and Manca, 2002), (Langton, 1986). More recently Kauffman (1999) introduced *boolean networks* as discrete paradigm for modeling the genetic regulatory mechanisms in the living cells. However, in the search for “grammars” of biological phenomena membrane systems seem to be a model that incorporates many crucial aspect that are typical of complex interactions.

The main difference between classical dynamical systems and string manipulating systems is the different perspective with respect to which they are considered. In fact, a grammar is a device used for generating languages, therefore in this sense what is important, in a process of grammatical derivation, is only its final product, that is, the string generated, rather than the “form” of the generation behavior. In other words, the processes are intended to have an end, those that are endless are neglected, and the final states of terminating processes are the objects defined by the grammar. But, imagine for a moment a “grammar” that is able to represent all the “states” of a living cell. In this

case all the ordinary processes are endless because termination implies death. Such a perspective changes radically the interest of study. Our investigation has initiated the analysis of discrete periodic phenomena. Many aspects seem to be very important for a future research. The following are some of them:

- The classification and the dependence of all possible behaviors in correspondence to some given initial situations and/or external perturbations.
- The articulation and harmonization of the life cycles internal to a system with the external environment cycle (this implies some kind of “internal time” and of synchronization tools).
- The stability of the system, and the coupling between cycles and irreversible processes.
- The decomposability of behaviors in components and the way they are interrelated.
- the conditions that ensure the life of the system in a given environment or in a specified class of environments;
- chaotic and involutive processes that determine terminating or “errating” states;
- the effect of one or more systems have in a given environment, and as a consequence, the process of formation of new stable environments.

## References

- Bernardini, F., 2002. *Analisi Dinamica di Sistemi a Membrane*. “Tesi di Laurea in Informatica”, Università di Pisa.
- Bernardini, F., Manca, V., 2002. P Systems with Boundary Rules. In *Preliminary Proceedings of Second Workshop on Membrane Computing*, Curtea de Arges, 97-101.
- Bonanno, C., Manca, V., 2002. Discrete dynamics in biological models. *Romanian Journal of Information Science and Technology*, **5**, (1-2), 45-67.
- Calude, C., Paun, Gh., 2002. *Computing with Cells and Atoms*. Taylor and Francis, London.
- Gheorghe, M. 2000. Generalized Stream X-Machines and Cooperating Distributed Grammar Systems. *Formal Aspects of Computing*, **12**, 459-472.
- Kauffman, S.A., 1999. *Investigations*. Oxford University Press.
- Langton C.G., 1986. Studying artificial life with cellular automata. *Physica D*, **22**, 120.
- Marcus, S., Păun, Gh., 1994. Infinite (almost periodic) words, formal languages and dynamical systems. *Bulletin EATCS*, **54**, 224.
- Von Neumann, J., 1966. *Theory of self-reproducing automata*. University of Illinois, Urbana.
- Păun, Gh., 2000. Computing with Membranes. *Journal of Computer and System Sciences*, **61**, 1, 108-143.
- Păun, Gh., 2001. From cells to computers: computing with membranes (P Systems). *BioSystems*, **59**, 139.
- Păun, A., Păun, Gh., 2002. The Power of Communication: P Systems with Symport/Antiport”. *New Generation Computer*, to appear.
- Rozenberg, G., Salomaa, A. (eds.), 1998. *Handbook of formal languages*. Springer, vol. 1-3.
- Suzuki, Y., Tanaka, H., 1997. Chemical Oscillation in symbolic chemical systems and its behavioral pattern. In Bar-Yam, Y. (ed.) *Proceedings of The International Conference on Complex Systems*, Nashua, NH.
- Zhabotinsky, A.M., 1991. A history of chemical oscillations and waves. *Chaos*, **1**, 379-386.