General multi-fuzzy sets and fuzzy membrane systems

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Abstract

We propose a certain fuzzification of membrane systems and their evolution rules which is motivated by some practical applications, where a strength (or weakness causing uncertainty) of an occurrence of an object in a system is determined not only by the number of occurring copies of that object but also by a quality of occurring copies.

1 Introduction

In the paper [5] A. Syropoulos introduced the concepts of general multi-fuzzy set and a fuzzy P system, where fuzzy P systems are fuzzy set-theoretic counterparts of P systems introduced by Gh. Păun(for basics of membrane computing, see [4]).

In the present paper we propose another approach to fuzzification of multisets and P systems, which is motivated by some practical applications in biochemistry and medical sciences, where a strength (or weakness causing uncertainty) of an occurrence of an object in a system is determined not only by the number of occurring copies of that object, but also by a quality of occurring copies.

The approach is aimed to prove:

- classification of processes generated by systems with respect to evolution rules transforming systems in the steps of processes,
- computer simulations of processes generated by application of evolution rules,
- analysis of evolution rules to simplify them (by idealization) for efficient computer simulation of processes.

In Section 2 we describe a new interpretation of general multi-fuzzy sets, different from that given in [5], which respects the strength of occurrence of objects in a system determined by the number and quality of occurring copies of these objects. Then in Sections 3, 4 we discuss some algebra of general multi-fuzzy sets focusing in their sum and their various subtractions used to describe an idea of an evolution rule of a general fuzzy membrane system introduced in Section 5.

The practical applications of the proposed approach will be reported in forthcoming papers concerning examples in biochemistry and medical sciences.

The author thanks Dr. J. Andrzej Pomykała for useful remarks and discussions.

2 Multisets, fuzzy sets, and general multi-fuzzy sets

In this section we recall the known concepts of multisets [3], fuzzy sets [1], and general multi-fuzzy sets [5]. We also present some new and practical interpretation of general multi-fuzzy sets.

Multisets over a set $\mathcal{O}$, called sometimes *Boolean multisets* over a set $\mathcal{O}$, are functions $M : \mathcal{O} \to \mathbb{N}$ valued in the set $\mathbb{N}$ of natural numbers, where $\mathcal{O}$ is a set of objects, and the value $M(x)$ is the number of copies of an object $x \in \mathcal{O}$ which (currently) occur in a system or its part. Thus multisets describe resources of copies of objects of $\mathcal{O}$. Characteristic functions of subsets of $\mathcal{O}$ are among multisets over $\mathcal{O}$. We write $\mathbb{N}^{\mathcal{O}}$ to denote the set of multisets over $\mathcal{O}$.

The usual orderings $<$, $\leq$ on $\mathbb{N}$, the usual operation $+$ of addition (sum) of natural numbers, and subtraction $\dot{-}$ of natural numbers given by

$$m \dot{-} n = \begin{cases} m - n & \text{if } m \geq n, \\ 0 & \text{otherwise,} \end{cases}$$

induce the orderings $<$, $\leq$, and the operations of addition (sum) $+$ and subtraction $\dot{-}$ of multisets which are defined *componentwise* by

$$\begin{aligned} & M_1 < M_2 \ [M_1 \leq M_2] \text{ iff } M_1(x) < M_2(x) \ [M_1(x) \leq M_2(x)] \text{ for all } x \in \mathcal{O}, \\ & (M_1 + M_2)(x) = M_1(x) + M_2(x), \\ & (M_1 \dot{-} M_2)(x) = M_1(x) \dot{-} M_2(x), \end{aligned}$$

for all $x \in \mathcal{O}$ and for all multisets M_1, M_2 over $\mathcal{O}$. We write 0 to denote that multiset M over $\mathcal{O}$ which is defined by

$$M(x) = 0 \text{ for all } x \in \mathcal{O}.$$

Fuzzy sets over a set $\mathcal{O}$ are functions $f : \mathcal{O} \to [0,1]$ valued in the unit interval $[0,1]$ of real numbers, where $\mathcal{O}$ is a set of *objects* and the value $f(x)$ is the degree of membership of an object $x \in \mathcal{O}$ in f.

Characteristic functions of subsets of $\mathcal{O}$ are among fuzzy sets over $\mathcal{O}$. The usual orderings $<$, $\leq$ of real numbers, and infima with suprema of finite sets of natural numbers in $[0,1]$ determine the orderings $<$, $\leq$ and the operations of union $\cup$ and intersection $\cap$ of fuzzy sets which are defined *pointwise* by

$$\begin{aligned} & f_1 < f_2 \ [f_1 \leq f_2] \text{ iff } f_1(x) < f_2(x) \ [f_1(x) \leq f_2(x)] \text{ for all } x \in \mathcal{O}, \\ & (f_1 \cup f_2)(x) = \sup\{f_1(x), f_2(x)\}, \\ & (f_1 \cap f_2)(x) = \inf\{f_1(x), f_2(x)\}, \end{aligned}$$

for all $x \in \mathcal{O}$ and for all fuzzy sets f_1, f_2 over $\mathcal{O}$. We write $[0,1]^{\mathcal{O}}$ to denote the set of fuzzy sets over $\mathcal{O}$ and we write 0 to denote that fuzzy set f over $\mathcal{O}$ which is defined by $f(x) = 0$ for all $x \in \mathcal{O}$.

General multi-fuzzy sets over a set $\mathcal{O}$ are functions $M : \mathcal{O} \times \mathbb{N} \to [0,1]$ or, equivalently, functions $\mathcal{M} : \mathcal{O} \to [0,1]^{\mathbb{N}}$, where $\mathcal{O}$ is a set of objects, $[0,1]$ is a unit interval of real numbers, $[0,1]^{\mathbb{N}}$ is the set of fuzzy sets of natural numbers, and the value $M(x,n)$ or $\mathcal{M}(x)(n)$ is the degree of certainty that n copies of an object $x \in \mathcal{O}$ occur in a system or its part. Since $[0,1]^{\mathbb{N}}$ is the set of fuzzy sets, the already defined orderings $<$, $\leq$, and operations of union and intersection of fuzzy sets of natural numbers induce the corresponding orderings and operations of general multi-fuzzy sets which are defined componentwise in a similar way as in the case of multisets.

We propose another interpretation of general multi-fuzzy sets which is motivated by some practical applications.

Since for a Boolean multiset $M : \mathcal{O} \to \mathbb{N}$ and an object $x \in \mathcal{O}$ the characteristic function of the segment $\{i \in \mathbb{N} | 0 \leq i \leq M(x)\}$ is among fuzzy sets of natural numbers and elements of this segment serve for numbering $M(x)$ copies of x such that a natural number i with $i < M(x)$ is identified with i-th copy of x, for a general multi-fuzzy set $\mathcal{M}$ over $\mathcal{O}$ and an object $x \in \mathcal{O}$ one can treate fuzzy set $\mathcal{M}(x)$ of natural numbers such that $\mathcal{M}(x)(i)$ is the degree of membership of i-th copy of x in $\mathcal{M}(x)$. One may claim here $\mathcal{M}(x)$ to be a fuzzy set theoretic counterpart of a Boolean segment of natural numbers (i.e., the set $\{i \in \mathbb{N} | 0 \leq i < n\}$ for $n \in \mathbb{N}$) in a more evident way.

We propose the following definition of fuzzy segments of natural numbers as the fuzzy set theoretic counterparts of Boolean segments of natural numbers.

By a *fuzzy segment* of natural numbers we mean a fuzzy set $f : \mathbb{N} \to [0,1]$ of natural numbers for which the following conditions hold:

(1) $m < n$ implies $f(m) \geq f(n)$ for all natural numbers m, n (*comonotonicity*),

(2) $f(k) = 0$ for some natural number k.

If $f = \mathcal{M}(k)$ for a multi-fuzzy set $\mathcal{M}$, condition (1) provides that there is no other (preference) relation between copies of x in f than that determined by their degree of membership in f and conditions (1) with (2) provide that there is only a finite number of copies of x with degree of membership greater than 0.

The following practical meaning can be given to the values $\mathcal{M}(x)(i)$ of a general multi-fuzzy set $\mathcal{M}$, where the values $\mathcal{M}(x)$ are fuzzy segments of natural numbers. If lifetime(x) is the average time of life of an object x and t_i is the current time counted from the birth (or emergence) of i-th copy of x, one can identify $\mathcal{M}(x)(i)$ with the number

$$1 \dot{-} \frac{t_i}{\text{lifetime}(x)}$$

which is the degree of freshness of i-th copy of x, where subtraction $\dot{-}$ of real numbers greater than 0 is defined in an analogous way as already defined subtraction of natural numbers. Hence the value $\mathcal{M}(x)(i)$ near 0 means that i-th copy of x tends to decay caused by aging and $\mathcal{M}(x)(i) = 0$ means the decay of i-th copy of x. Therefore $\mathcal{M}(x)$ describes the current freshness of copies of x occurring in a system specified by the values $\mathcal{M}(x)(i)$.

The above practical meaning of the values $\mathcal{M}(x)(i)$ was suggested to the author by the discussion of fuzzy timed Petri nets in [4] and by the lecture [6].

There are other possibilities of assigning practical meaning to the values $\mathcal{M}(x)(i)$ of a general multi-fuzzy set $\mathcal{M}$ over $\mathcal{O}$ with $\mathcal{M}(x)$ being fuzzy segments of natural numbers, for instance $\mathcal{M}(x)(i)$ can be a relative amount of energy carried by i-th copy of x.

Thus general multi-fuzzy set $\mathcal{M}$ over $\mathcal{O}$ with values $\mathcal{M}(x)$ being fuzzy segments of natural numbers can describe a strength of occurrence of objects which is determined by the quality of currently occurring copies of objects.

In the next two sections we describe some useful properties of fuzzy segments of natural numbers.

3 Properties of fuzzy segments of natural numbers

One can represent fuzzy segments of natural numbers by *finite multisets* over $(0,1]$ which are defined to be such that $\{\alpha \in (0,1] \,|\, M(\alpha) > 0\}$ is a finite set, where $(0,1]$ is left open unit interval of real numbers, i.e., $(0,1] = \{x \in \mathbb{R} \,|\, 0 < x \leq 1\}$ for $\mathbb{R}$ denoting the set of real numbers. Let $\mathrm{SGM}(\mathbb{N},[0,1])$ denote the set of fuzzy segments of natural numbers and let $\mathrm{FIN}((0,1],\mathbb{N})$ denote the set of finite multisets over $(0,1]$.

A representation of fuzzy segments of natural numbers by finite multisets over $(0,1]$ is provided by a mapping $(-)^{\S} : \mathrm{SGM}(\mathbb{N},[0,1]) \to \mathrm{FIN}((0,1],\mathbb{N})$ defined in the following way for every fuzzy segment f of natural numbers:

$$(f)^{\S}(\alpha) \text{ is the number of elements of the set } \{i \in \mathbb{N} \,|\, f(i) = \alpha\}$$

for all $\alpha \in (0,1]$.

Proposition 3.1 *The mapping $(-)^{\S}$ is a bijection whose inverse*

$$(-)^{-\S} : \mathrm{FIN}((0,1],\mathbb{N}) \to \mathrm{SGM}(\mathbb{N},[0,1])$$

is defined in the following way for every finite multiset M over $(0,1]$:

- *if $\alpha \in (0,1] \,|\, M(\alpha) > 0\}$ is empty, then $(M)^{-\S}(n) = 0$ for all natural numbers n,*
- *if $\alpha \in (0,1] \,|\, M(\alpha) > 0\}$ is nonempty, then its elements form finite decreasing string $\alpha_0 > \ldots > \alpha_{k-1}$ of real numbers for k equal to the number of elements of $\{\alpha \in (0,1] \,|\, M(\alpha) > 0\}$ and one defines*

$$(M)^{-\S}(i) = \begin{cases} \alpha_0 & \text{if } 0 \leq i \leq M(\alpha_0), \\ \alpha_j & \text{if } \sum_{m=0}^{j-1} M(\alpha_m) \leq i < \sum_{m=0}^{j} M(\alpha_m) \text{ and } 0 < j < k, \\ 0 & \text{otherwise.} \end{cases}$$

Proof. We prove that $((f)^{\S})^{-\S} = f$ and $((M)^{-\S})^{\S} = M$ in an immediate way by using the definitions of $(-)^{\S}$ and $(-)^{-\S}$. ∎

Therefore the mappings $(-)^{\S}$ and $(-)^{-\S}$ provide a representation of fuzzy segments of natural numbers by finite multisets over $(0,1]$.

Thus one simply defines the sum $+$ of fuzzy segments f, g of natural numbers by

$$f + g = ((f)^{\S} + (g)^{\S})^{-\S},$$

where $+$ standing in the right hand side of the above equation is the sum of Boolean multisets already defined.

Lemma 3.1 *For two finite multisets M_1, M_2 over $(0,1]$ the inequality $M_1 \leq M_2$ implies the inequality $(M_1)^{-\S} \leq (M_2)^{-\S}$.*

Proof. We prove the lemma by induction on n equal to the number of the elements of the set $\{\alpha \in (0,1] \,|\, M_1(\alpha) > 0\}$. ∎

There are simple examples showing that $(M_1)^{-\S} \leq (M_2)^{-\S}$ does not imply $M_1 \leq M_2$.

By analogy with sum of fuzzy segments of natural numbers one defines subtraction $\dot{-}$ of fuzzy segments f, g of natural numbers by

$$f \dot{-} g = ((f)^{\S} \dot{-} (g)^{\S})^{-\S},$$

where $\dot{-}$ standing in the right hand side of the above equation is subtraction of Boolean multisets already defined.

For a fuzzy segment f of natural numbers we define

$$\text{size}(f) = \min\{i \in \mathbb{N} \,|\, f(i) = 0\}.$$

Lemma 3.2 *For all fuzzy segments f, g of natural numbers the inequality $(f)^{\S} \geq (g)^{\S}$ implies the following conditions:*

(a) $\text{size}(f \dot{-} g) = \text{size}(f) - \text{size}(g)$,

(b) $(f \dot{-} g) + g = f$.

Proof. The lemma is an immediate consequence of the definitions of $+$ and $\dot{-}$ of fuzzy segments. ∎

We use the following property of subtraction of fuzzy segments of natural numbers.

For a fuzzy segment f of natural numbers and an integer $k \geq -1$ we define a fuzzy segment $f \upharpoonright k$ of natural numbers by

$$(f \upharpoonright k)(i) = \begin{cases} f(i) & \text{if } 0 \leq i \leq k, \\ 0 & \text{if } k = -1 \text{ and } i \geq 0, \\ 0 & \text{if } i > k > -1. \end{cases}$$

Lemma 3.3 *For all fuzzy segments f of natural numbers and all integers $k \geq -1$ the following condition holds:*

$$(f \dot{-} (f \upharpoonright k))(i) = f(i + k + 1) \quad \textit{for every } i \in \mathbb{N}.$$

Proof. The lemma is an immediate consequence of the definitions of subtraction of fuzzy segments and $f \restriction k$. ■

Since in general $f \geq g$ does not imply $(f)^{\S} \geq (g)^{\S}$ for fuzzy segments f and g of natural numbers, it remains to discuss possibilities of subtracting fuzzy segment of natural numbers which respect the pointwise defined ordering $\leq$ of fuzzy segments of natural numbers; a counterpart of Lemma 3.2 with $(f)^{\S} \geq (g)^{\S}$ replaced by $f \geq g$ holds for that subtracting.

4 Subtracting fuzzy segments with respect to their pointwise defined ordering $\leq$

For two fuzzy segments f, g of natural numbers with $f \geq g$ we say that a finite multiset M over $(0, 1]$ is a *subtractive choice multiset* with respect to $f \geq g$ if the following conditions hold:

(i) $M \leq (f)^{\S}$,

(ii) $g \leq (M)^{-\S}$,

(iii) $\mathrm{size}((M)^{-\S}) = \mathrm{size}(g)$.

A subtractive choice multiset M with respect to $f \geq g > 0$ represents the chosen numbers of copies of real numbers in $(0, 1]$ to be deleted (subtracted) from $(f)^{\S}$ providing (ii) and (iii).

Thus for two fuzzy segments f, g of natural numbers with $f \geq g$ and subtractive choice multiset M with respect to $f \geq g$ we define *subtraction of f and g determined by M* to be that fuzzy segment $f \,\$_M\, g$ of natural numbers which is given by

$$f \,\$_M\, g = f \dot{-} (M)^{-\S}.$$

Lemma 4.1 *For every subtractive choice multiset M with respect to $f \geq g$ the following conditions hold*:

(a′) $\mathrm{size}(f \,\$_M\, g) = \mathrm{size}(f) - \mathrm{size}(g)$,

(b′) $(f \,\$_M\, g) + (M)^{-\S} = f$.

Proof. The lemma is a consequence of Lemma 3.2 and of the definition of $f \,\$_M\, g$. ■

For two fuzzy segments f, g of natural numbers with $f \geq g > 0$ we define

$$\Phi_{f,g} = \{(M)^{-\S} \mid M \text{ is a subtractive choice multiset with respect to } f \geq g\}.$$

We show that $\Phi_{f,g}$ has the greatest element and the smallest element with respect to pointwise ordering $\leq$ of fuzzy segments of natural numbers.

Proposition 4.1 *Let f, g be two fuzzy segments of natural numbers with $f \geq g > 0$. Then $f \restriction (\mathrm{size}(g) - 1)$ is the greatest element in the set $\Phi_{f,g}$ with respect to pointwise defined ordering of fuzzy segments of natural numbers.*

*Proof.*Since by Lemma 3.1 $(h)^{\S} \leq (h')^{\S}$ implies $h \leq h' \restriction (\text{size}(h) - 1)$ for all fuzzy segments h and h', we obtain by Lemma 3.1 and conditions (i), (iii) the following inequality

$$(M)^{-\S} \leq f \restriction (\text{size}(g) - 1)$$

for all subtractive choice multisets M with respect to $f \geq g > 0$. Therefore $f \restriction (\text{size}(g) - 1)$ is the greatest element in $\Phi_{f,g}$ with respect to pointwise defined ordering $\leq$ of fuzzy segments of natural numbers. ■

A fuzzy segment g of natural numbers is called a *threshold fuzzy segment* if the set $\{g(i) \,|\, i \in \mathbb{N} \text{ and } 0 \leq i < \text{size}(g)\}$ has at most one element which we call *threshold value* of g.

Proposition 4.2 *Let f, g be two fuzzy segments with $f \geq g > 0$ such that g is a threshold fuzzy segment. Then $(f \mathbin{\dot{-}} (f \restriction c)) \restriction (\text{size}(g) - 1)$ is the smallest element in $\Phi_{f,g}$ with respect to the pointwise defined ordering $\leq$ of fuzzy segments of natural numbers, where*

$$c = \max(\{j \in \mathbb{N} | \, f \mathbin{\dot{-}} (f \restriction j) \geq g\} \cup \{-1\}).$$

*Proof.*For $c = -1$ we have $\Phi_{f,g} = \{f \restriction (\text{size}(g) - 1)\}$, hence the proposition is true. For $c \geq 0$ let $(f \mathbin{\dot{-}} (f \restriction c)) \restriction (\text{size}(g) - 1)$ be denoted by f^*. We show that the negation of the inequality

$$(+) \qquad f^* \leq (M)^{-\S}$$

leads to a contradiction for a subtractive choice multiset M with respect to $f \geq g > 0$. Since the negation of (+) means that

$$f^*(j) > (M)^{-\S}(j) \text{ for some } j \text{ with } 0 \leq j < \text{size}(g),$$

we obtain by (i) and (ii) holding for M and by comonotonicity of $(M)^{-\S}$ that the following strong inequality holds

$$\sum_{x \in D(j)} M(x) < \sum_{x \in D(j)} (f^*)^{\S}(x)$$

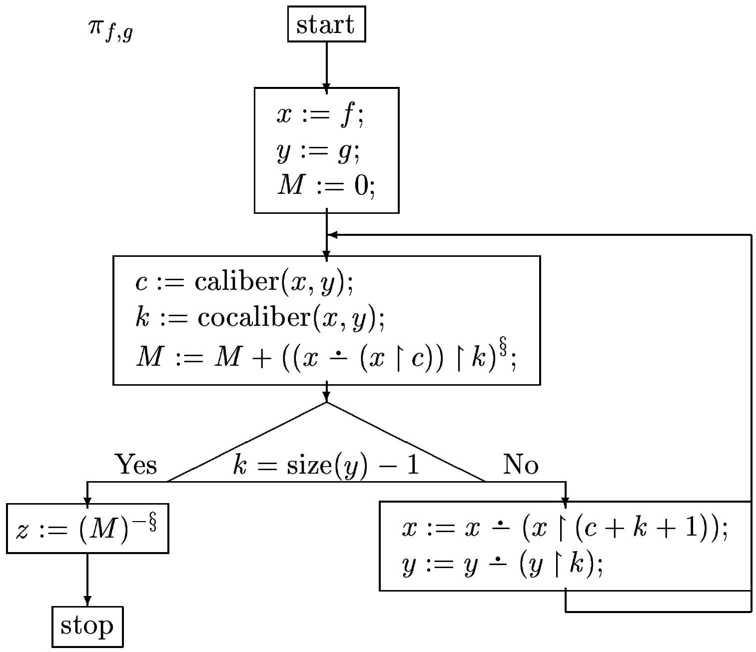
for $D(j) = \{x \in (0,1] \,|\, x \leq (M)^{-\S}(j) \text{ and } 0 < \max\{M(x), (f^*)^{\S}(x)\}\}$ and for some natural number j with $0 \leq j < \text{size}(g) - 1$. Hence condition (iii) does not hold for M. Therefore the negation of the inequality (+) leads to a contradiction. Thus f^* is the smallest element in $\Phi_{f,g}$ with respect to pointwise defined ordering $\leq$ of fuzzy segments of natural numbers. ■

A generalization of Proposition 4.2 to arbitrary fuzzy segments f, g of natural numbers with $f \geq g > 0$ requires more considerations.

For two fuzzy segments $f \geq g > 0$ of natural numbers we define:

$$\begin{aligned}
\text{caliber}(f,g) &= \max(\{j \in \mathbb{N} | \, f \mathbin{\dot{-}} (f \restriction j) \geq g\} \cup \{-1\}), \\
\text{cocaliber}(f,g) &= \min\Big(\{k \in \mathbb{N} | \, g \mathbin{\dot{-}} (g \restriction k) > 0 \text{ and} \\
&\qquad \text{caliber}(f \mathbin{\dot{-}} (f \restriction (\text{caliber}(f,g) + k + 1)), g \mathbin{\dot{-}} (g \restriction k)) \geq 0\} \\
&\qquad \cup \{\text{size}(g) - 1\}\Big).
\end{aligned}$$

The procedure $\pi_{f,g}$ described by the diagram below is aimed to compute the smallest element in the set $\Phi_{f,g}$ with respect to pointwise defined ordering $\leq$ of fuzzy segments of natural numbers.



Theorem 4.1 *For all fuzzy segments $f \geq g > 0$ of natural numbers the fuzzy segment z resulting from the procedure $\pi_{f,g}$ is the smallest element in the set $\Phi_{f,g}$ with respect to the pointwise defined ordering of fuzzy segments of natural numbers.*

Proof. We can prove the theorem by induction on the number n of elements of the set $\{g(i) \,|\, i \in \mathbb{N} \text{ and } g(i) > 0\}$. For the case $n = 1$ the proof is analogous to the proof of Proposition 4.2. ∎

For two fuzzy segments f, g of natural numbers with $f \geq g$ we define the *high subtraction* $f \,\$^{\top}\, g$ of $f \geq g$ and the *low subtraction* $f \,\$^{\perp}\, g$ of $f \geq g$ by

$$f \,\$^{\top}\, g = f \,\$_M\, g \quad \text{for} \quad (M)^{-\S} = f \upharpoonright (\text{size}(g) - 1)$$

and

$$f \,\$^{\perp}\, g = \begin{cases} f & \text{if size}(g) = 0, \\ f \,\$_{\mathbf{Z}}\, g & \text{otherwise,} \end{cases}$$

where $(\mathbf{Z})^{-\S}$ is the smallest element in $\Phi_{f,g}$ with respect to pointwise defined ordering $\leq$ of fuzzy segments or, equivalently, $(\mathbf{Z})^{-\S}$ is equal to the result of the procedure $\pi_{f,g}$.

Let $\mathcal{M}$ and $\mathcal{M}'$ be two general multi-fuzzy sets over $\mathcal{O}$ whose values $\mathcal{M}(x)$ and $\mathcal{M}'(x)$ are fuzzy segments of natural numbers and let $\mathcal{M} \geq \mathcal{M}'$, i.e., $\mathcal{M}(x) \geq \mathcal{M}'(x)$

for all $x \in \mathcal{O}$. Thus for a function $\delta : \mathcal{O} \to \{\bot, \top\}$ one defines the subtraction $\mathcal{M} \,\$^{\delta}\, \mathcal{M}'$ of $\mathcal{M} \geq \mathcal{M}'$ to be a general multi-fuzzy set defined componentwise by

$$(\mathcal{M} \,\$^{\delta}\, \mathcal{M}')(x) = \mathcal{M}(x) \,\$^{\delta(x)}\, \mathcal{M}'(x) \quad \text{for all } x \in \mathcal{O}.$$

According to the interpretation (given in Section 2) of general multi-fuzzy sets $\mathcal{M}$ with values $\mathcal{M}(x)$ being fuzzy segments of natural numbers we explain the subtraction $\mathcal{M} \,\$^{\delta}\, \mathcal{M}'$ of $\mathcal{M} \geq \mathcal{M}'$ in the following way.

If $\delta(x) = \top$ and $\text{size}(\mathcal{M}(x)) > \text{size}(\mathcal{M}'(x)) > 0$, the value $(\mathcal{M} \,\$^{\delta}\, \mathcal{M}')(x) = \mathcal{M}(x) \,\$^{\top}\, \mathcal{M}'(x)$ is the result of deletion from $\mathcal{M}(x)$ the number $\text{size}(\mathcal{M}'(x))$ of those copies of x which are of relatively high degree of membership in $\mathcal{M}(x)$ by means their degrees $\mathcal{M}(x)(i)$ of membership are not smaller than $\mathcal{M}(x)(\text{size}(\mathcal{M}'(x)) - 1)$.

If $\delta(x) = \bot$ and $\text{size}(\mathcal{M}(x)) > \text{size}(\mathcal{M}'(x)) > 0$, the value $(\mathcal{M} \,\$^{\delta}\, \mathcal{M}')(x) = \mathcal{M}(x) \,\$^{\bot}\, \mathcal{M}'(x)$ is the result of deletion from $\mathcal{M}(x)$ of the number $\text{size}(\mathcal{M}'(x))$ of those copies of x which are of relatively low degree of membership in $\mathcal{M}(x)$ by means that they form the smallest fuzzy segment z in $\Phi_{\mathcal{M}(x),\mathcal{M}'(x)}$, i.e., z is the smallest fuzzy segment such that $\text{size}(z) = \text{size}(\mathcal{M}'(x))$ and $z \geq \mathcal{M}'(x)$.

5 General fuzzy membrane systems and their evolution rules

In this section we introduce the concept of a general fuzzy membrane system and then present some evolution rules of general fuzzy membrane systems.

We recall according to [2] and [3] that a (*Boolean*) *membrane system* $\mathcal{S}$ is given by the following data:

- a finite non-empty set $\mathbb{B}_{\mathcal{S}}$ of balls of finite diameters greater than 0 (in Euclidean space E^n for $n \geq 1$) such that the frontiers of the balls contained in $\mathbb{B}_{\mathcal{S}}$ are pairwise disjoint sets, and there exists the greatest ball b_0 in $\mathbb{B}_{\mathcal{S}}$ with respect to the inclusion relation $\subseteq$; the balls contained in $\mathbb{B}_{\mathcal{S}}$ are called *membranes* of $\mathcal{S}$ and $\mathbb{B}_{\mathcal{S}}$ is called the *underlying set of membranes* of $\mathcal{S}$;

- three functions $l_{\mathcal{S}} : \mathbb{B}_{\mathcal{S}} \to L_{\mathcal{S}}$, $e_{\mathcal{S}} : \mathbb{B}_{\mathcal{S}} \to \{-, 0, +\}$, $\mathcal{M}_{\mathcal{S}} : \mathcal{O}_{\mathcal{S}} \to N^{\mathcal{O}_{\mathcal{S}}}$, where $L_{\mathcal{S}}$ is the *set of labels* of $\mathcal{S}$, $l_{\mathcal{S}}$ is called the *labelling function* of $\mathcal{S}$, $e_{\mathcal{S}}$ is called the *electric charge function* of $\mathcal{S}$, and $\mathcal{M}_{\mathcal{S}}$, called the *object distribution function* of $\mathcal{S}$, is a function whose values $\mathcal{M}_{\mathcal{S}}(m)$ are Boolean multiset over $\mathcal{O}_{\mathcal{S}}$ for $\mathcal{O}_{\mathcal{S}}$ called the *set of objects* of $\mathcal{S}$.

The value $l_{\mathcal{S}}(m)$ is the label assigned to a membrane m of $\mathcal{S}$, $e_{\mathcal{S}}(m)$ is the electric charge of a membrane m of $\mathcal{S}$, and the value $\mathcal{M}_{\mathcal{S}}(m)$ is a Boolean multiset M whose values $M(x)$ are the numbers of copies of $x \in \mathcal{O}_{\mathcal{S}}$ contained in the *region of a membrane* m of $\mathcal{S}$ which is the space between the frontier of m and the frontiers of those membranes of $\mathcal{S}$ which are immediate subsets of m.

A *general fuzzy membrane system* $\mathcal{S}$ is defined in an analogous way as a Boolean membrane system except the object distribution function $\mathcal{M}_{\mathcal{S}}$ of $\mathcal{S}$ is such that the values $\mathcal{M}_{\mathcal{S}}(m)$ for $m \in \mathbb{B}_{\mathcal{S}}$ are those general multi-fuzzy sets $\mathcal{M}$ over $\mathcal{O}_{\mathcal{S}}$ whose values $\mathcal{M}(x)$ are fuzzy segments of natural numbers.

We consider evolution rules of general fuzzy membrane systems which are expressions of the form

$$(*) \qquad [_h\mathcal{N} \to \delta, \varepsilon, \mathcal{N}']_h^{\alpha},$$

where $\mathcal{N}$, $\mathcal{N}'$ are general multi-fuzzy sets over a set $\mathcal{O}$ (or some presentations for them) whose values $\mathcal{N}(x), \mathcal{N}'(x)$ are fuzzy segments of natural numbers ($x \in \mathcal{O}$), h is a label in a set L of labels, $\alpha \in \{-, 0, +\}$, δ is a function defined on $\mathcal{O}$ and valued in $\{\bot, \top\}$, and $\varepsilon \in [0, 1]$. We assume that $\mathcal{N}'(x)(i) = 1$ for all natural numbers i with $0 \leq i < \text{size}(\mathcal{N}'(x))$ and for $\mathcal{N}'(x) > 0$. The evolution rules of the form $(*)$ have common features with some evolution rules in [2] and [3].

For a general fuzzy membrane system $\mathcal{S}$ and an evolution rule R of the form $(*)$ with $\mathcal{O} = \mathcal{O}_{\mathcal{S}}$ and $L = L_{\mathcal{S}}$ we say that this rule R *can be applied to a membrane* m *of* $\mathcal{S}$ if the following conditions hold:

- $l_{\mathcal{S}}(m) = h$ and $e_{\mathcal{S}}(m) = \alpha$,
- for $\mathcal{M} = \mathcal{M}_{\mathcal{S}}(m)$ we have $\mathcal{M} \geq \mathcal{N}$, i.e. $\mathcal{M}(x) \geq \mathcal{N}(x)$ for all objects x of $\mathcal{S}$, where $\geq$ occurring in the last inequality is the pointwise defined ordering of fuzzy segments of natural numbers.

The *result of the application* of R to a membrane m of $\mathcal{S}$, whenever R can be applied to m, is a general fuzzy membrane system $\mathcal{S}'$ which is the same as $\mathcal{S}$ except

$$\mathcal{M}_{\mathcal{S}'}(m) = ((\mathcal{M}_{\mathcal{S}}(m) \,\$^{\delta}\, \mathcal{N}) \div \varepsilon) + \mathcal{N}',$$

where for a general multi-fuzzy set $\mathcal{M}$ over $\mathcal{O}$ and $\varepsilon \in [0, 1]$ we define $(\mathcal{M} \div \varepsilon)(x)(i) = \mathcal{M}(x)(i) \dot{-} \varepsilon$ for all objects $x \in \mathcal{O}$ and natural numbers $i \in \mathbb{N}$. Here ε is meant to be an average expense of time (or energy, etc.) consumed during the application of the rule.

Other evolution rules of general fuzzy membrane systems will be discussed in a forthcoming papers about some practical applications of the approach proposed in the present paper.

An application of an evolution rule of the form $(*)$ to a membrane of a general fuzzy membrane system and the result of this application can be explained from the point of view of practical applications in the following way.

An emergence of new high quality (fresh) copies (presented by $\mathcal{N}'$ in $(*)$) in the region of a membrane causes the deletion from this region of the old copies forming those segments above the segments presented by $\mathcal{N}$ in $(*)$ (i.e., those segments $\geq \mathcal{N}(x)$ for objects $x \in \mathcal{O}_{\mathcal{S}}$) which according to the function δ in $(*)$ are relatively high or relatively low (see the explanation of $\mathcal{M} \,\$^{\delta}\, \mathcal{M}'$ given in Section 4). An emergence of new copies of objects can be meant here in two ways such that new copies have been introduced (maybe from outside) to eliminate (to delete) some old copies or new copies have been generated by consuming (deleting) some old copies. The first of the above meanings of emergence of new copies has been suggested by [6], namely one can treat new copies as copies of some enzymes introduced to a system by some medicine aimed to eliminate simultaneously the old copies of enzymes according to the general multi-fuzzy set $\mathcal{N}$ and the function δ in $(*)$.

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